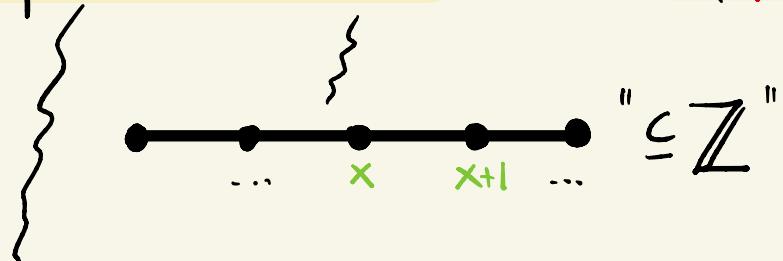


Dimerisation in quantum spin chains with $O(n)$ symmetry

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joint work with { J. Björnberg
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D. Ueltschi

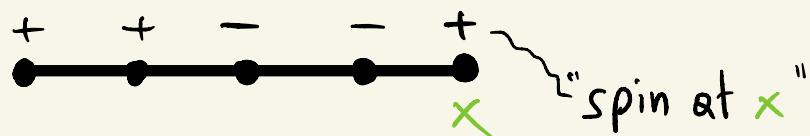
quantum spin $S \in \frac{1}{2}\mathbb{N}$ chain with $O(n=2S+1)$ symmetry



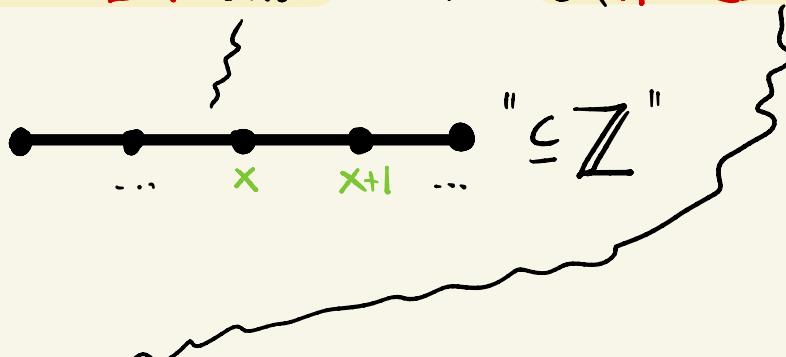
to each site x
attach & copy of \mathbb{C}^{2S+1} ($= \mathbb{C}^n$)

e.g. $S = \frac{1}{2} \Rightarrow n = 2$ and state space = $\bigotimes_{\mathbf{x}} \mathbb{C}^2$

(local) basis: $|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



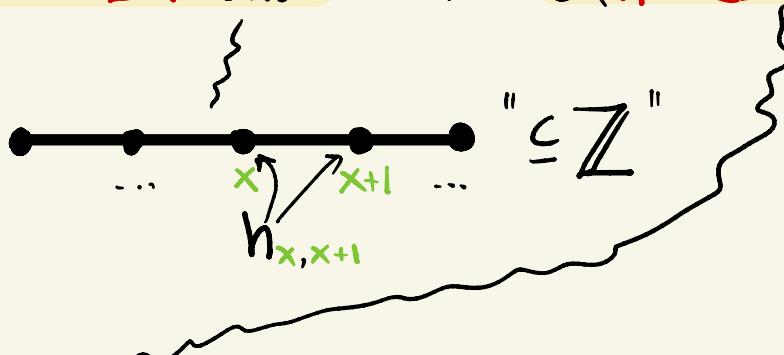
quantum spin $S \in \frac{1}{2}\mathbb{N}$ chain with $O(n=2S+1)$ symmetry



Hamiltonian H with state space $= \bigotimes_{\text{green } x} \mathbb{C}^n$

such that \forall orthogonal $O : HO = OH$

quantum spin $S \in \frac{1}{2} \mathbb{N}$ chain with $O(n=2S+1)$ symmetry



Hamiltonian H with state space $= \bigotimes C^n$

such that \forall orthogonal $O : HO = OH$

lemma $H = \sum_x h_{x,x+1}$ as above $\Rightarrow \exists u, v \in \mathbb{R} : h = uT + vQ$ (+c)

translation-invariant
nearest neighbour interactions

$$T |a\rangle \otimes |b\rangle = |b\rangle \otimes |a\rangle$$

$$Q = |\Psi\rangle \langle \Psi|$$

$$\text{for } \Psi = \frac{1}{\sqrt{n}} \sum_{i=1}^n e_i \otimes e_i \text{ ONB}$$

our model:

$$H_\ell = - \sum_{x=-\ell+1}^{\ell-1} \nu T_{x,x+1} + \cancel{\nu Q_{x,x+1}} \quad (\nu=1)$$

equilibrium states: $\langle \cdot \rangle_{B,\ell} = \frac{\text{Tr}(\cdot e^{-B H_\ell})}{\text{Tr}(e^{-B H_\ell})}$ $B \rightarrow \infty$

main result (B.M.N.U. '21) $\exists n_0: \forall n > n_0, |t| < n^{-4/3}$

$\lim_{\ell \rightarrow \infty} \lim_{B \rightarrow \infty} (-1)^\ell (\langle R_{-1,0} \rangle_{B,\ell} - \langle R_{0,1} \rangle_{B,\ell}) > 0$, "dimerisation"

where $R_{xy} = T_{xy} + n Q_{xy}$.



- also:
- H_ℓ has a positive spectral gap (uniform in ℓ)
 - exponential decay of spin-spin correlations

Why care about...

...dimerisation?

Q: is there a unique ∞ -volume ground state $\langle \cdot \rangle$?

💡: Q $\Leftrightarrow \lim_{\Lambda \uparrow \mathbb{Z}} \langle R \rangle_\Lambda$ exists & local R, independent of $\Lambda \uparrow \mathbb{Z}$

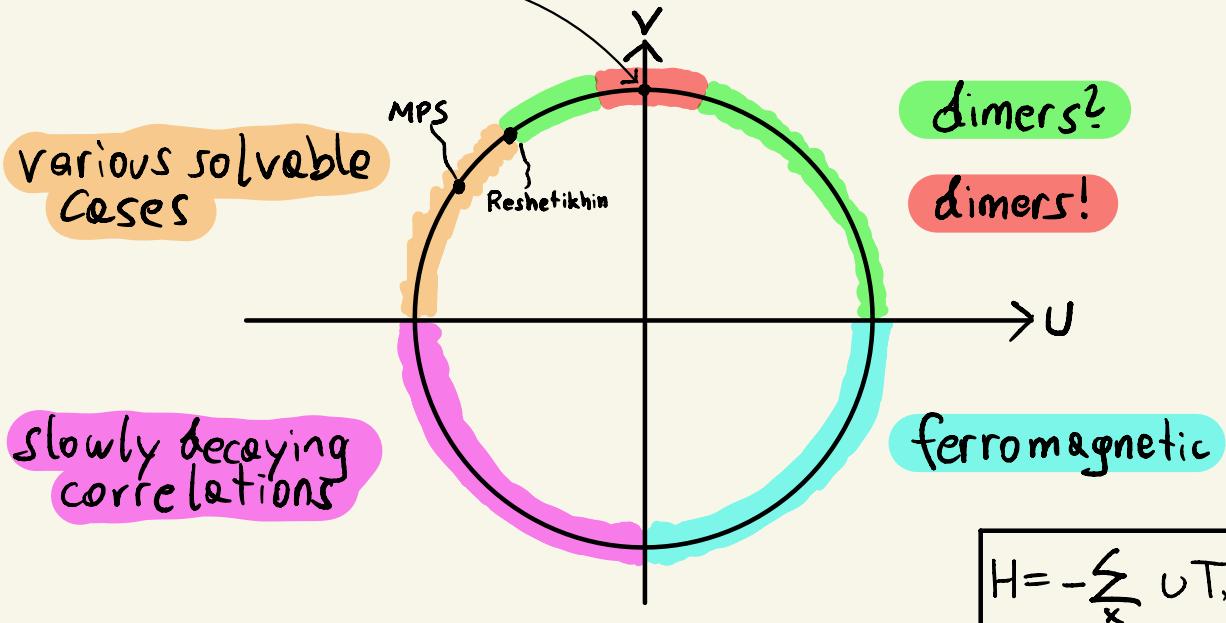
"A": dimerisation $\Rightarrow \exists R: \lim_{\Lambda \uparrow \mathbb{Z}} \langle R \rangle_\Lambda$ depends on Λ

$\Rightarrow \nexists$ ∞ -volume ground state
due to broken translation invariance

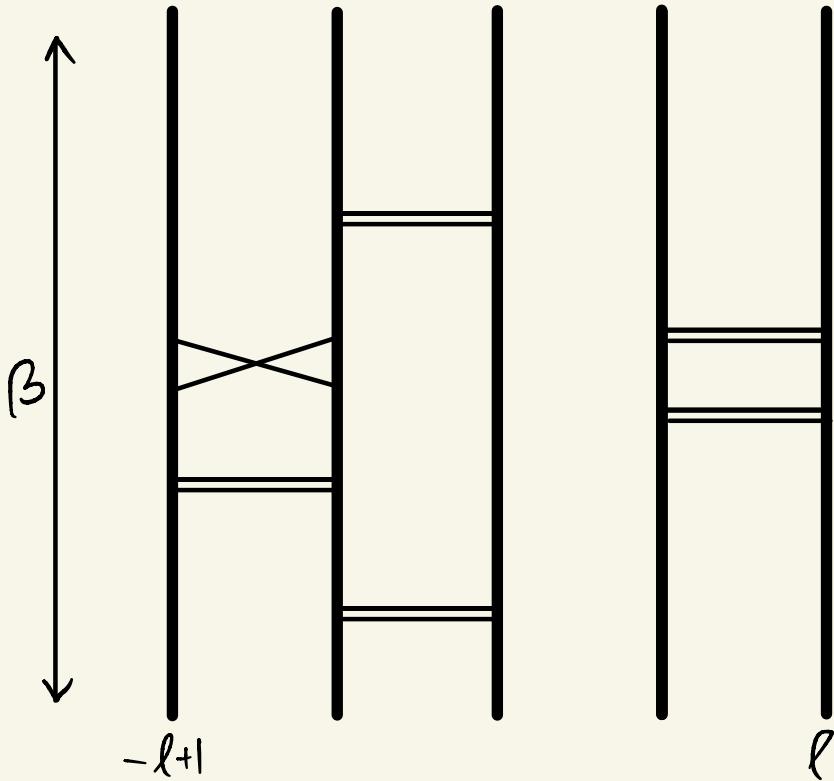
Why care about...

...our model / result?

- rich ground state phase diagram
- well-studied



Probabilistic representation (Feynman-Kac like)

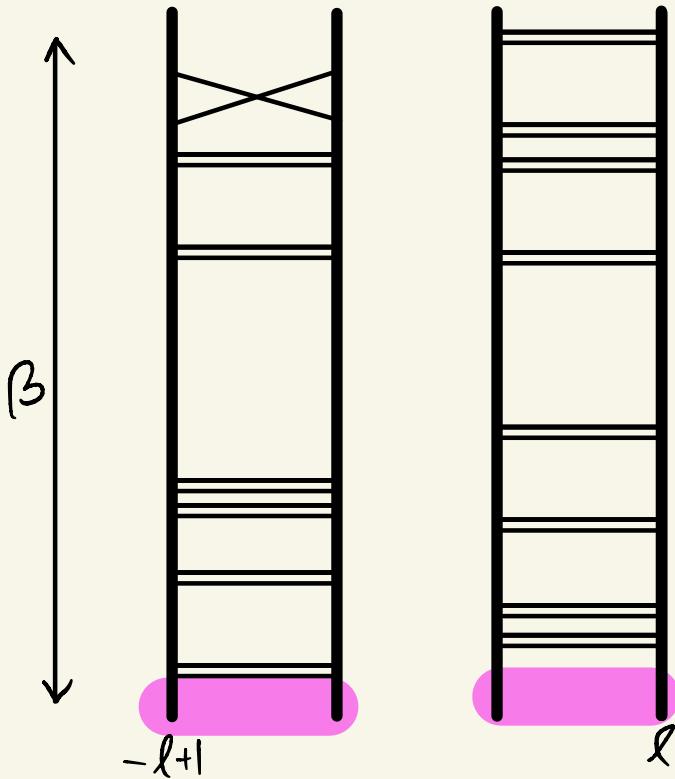


- Tóth '93
- Aizenman-Nachtergaele '94
- Ueltschi '13

- 1) place $\overbrace{}$ and $\diagup \diagdown$ at $\{x, x+l\}, t$ w.p. dt
- 2) weigh each config by $\propto U^{\# \times} n^{\# \text{loops} - \# \overbrace{}}$

Lemma $\langle R_{xy} \rangle_{\beta, 0} = P(\overbrace{x \cdots y})$
! signed measure if $U < 0$

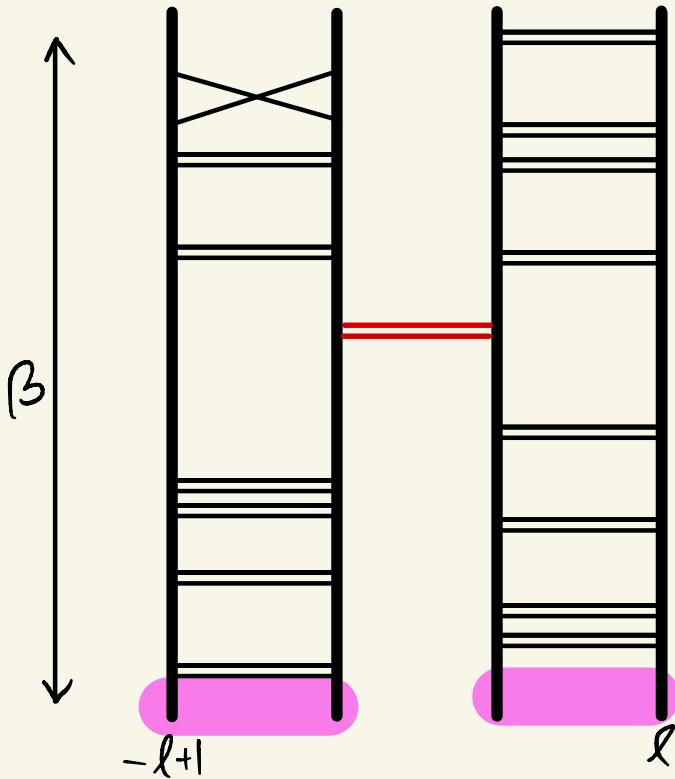
"optimal" configurations



- "maximises" $U \propto n^{\# \text{loops} - \# \text{excitations}}$ for $n \gg 1$, $|U| \ll 1$
- long loops = excitations
 - obtain a contour model
 - use CLUSTER EXPANSION to perturb around optimal configs.

necessary conditions hard to prove

"optimal" configurations



- "maximises" $U \propto n^{\# \text{loops} - \frac{1}{2}}$
for $n \gg 1$, $|l| \ll 1$
- long loops = excitations
 - obtain a contour model
 - use CLUSTER EXPANSION to perturb around optimal configs.

necessary conditions hard to prove



arXiv:2101.11464

Thanks!

