Bogoliubov Theory for Many-Body Quantum Systems

Below I list some references related to my classes. The list is of course not complete and mostly contains papers that I think are closest to the ideas I presented.

First of all, this is the original paper of Bogoliubov:

N. N. Bogoliubov. On the theory of superfluidity. *Izv. Akad. Nauk. USSR* **11** (1947), 77. Engl. Transl. *J. Phys. (USSR)* **11** (1947), 23.

Spectrum of Bose gases in the mean field limit

In my first class I explained how Bogoliubov theory can be used to estimate the ground state energy and to determine the low-energy excitation spectrum of a Bose gas in the mean field limit. Some papers addressing this problem are:

R. Seiringer. The Excitation Spectrum for Weakly Interacting Bosons. *Comm. Math. Phys.* **306** (2011), 565—578. (This paper deals with particles on the torus).

P. Grech, R. Seiringer. The excitation spectrum for weakly interacting bosons in a trap. *Comm. Math. Phys.* **322** (2013), no. 2, 559-591. (Particles trapped by an external potential).

M. Lewin, P. T. Nam, S. Serfaty, J.P. Solovej. Bogoliubov spectrum of interacting Bose gases. *Comm. Pure Appl. Math.* **68** (2014), 3, 413 – 471. (This paper follows the approach I presented).

J. Dereziński, M. Napiórkowski. Excitation Spectrum of Interacting Bosons in the Mean-Field Infinite-Volume Limit. *Annales Henri Poincaré* **15** (2014), 2409-2439. (Alternative approach).

The following papers give instead expansions of the energy in inverse powers of N:

A. Pizzo. Bose particles in a box III. A convergent expansion of the ground state of the Hamiltonian in the mean field limiting regime. Preprint arxiv:1511.07026.

L. Bossmann, S. Petrat, R. Seiringer. Asymptotic expansion of low-energy excitations for weaklyinteracting bosons. Preprint arXiv:2006.09825.

Dynamics of bosons in mean-field limit

In my second class, I discussed dynamics of mean-field bosons. There is a very large literature on this topic. Here, I only list some papers that focus on Bogoliubov theory and on fluctuations around the Hartree dynamics. The first works in this direction are:

K. Hepp. The classical limit for quantum mechanical correlation functions. *Comm. Math. Phys.* **35** (1974), 265–277.

J. Ginibre and G. Velo. The classical field limit of scattering theory for nonrelativistic many-boson systems. I and II. *Comm. Math. Phys.* **66** (1979), no. 1, 37–76, and **68** (1979), no. 1, 45–68.

The approach I followed in class was taken from:

M. Lewin, P. T. Nam, B. Schlein. Fluctuations around Hartree states in the mean-field regime. *Am. J. Math.* **137** (2015), no. 6, 1613–1650.

Some other recent works are:

M. Grillakis, M. Machedon and D. Margetis. Second-order corrections to mean-field evolution of weakly interacting bosons. I. *Comm. Math. Phys.* **294** (2010), no. 1, 273–301.

D. Mitrouskas, S. Petrat, P. Pickl. Bogoliubov corrections and trace norm convergence for the Hartree dynamics. *Rev. Math. Phys.* **31** (2019), no. 8.

Norm approximations for the time evolution have been used to show a central limit theorem and, very recently, a large deviation principle for quantum dynamics. This is the content of the following works:

S. Buchholz, C. Saffirio, B. Schlein. Multivariate central limit theorem in quantum dynamics. *J. Stat. Phys.* **154** (2014), no. 1-2, 113–152.

K. Kirkpatrick, S. Rademacher, B. Schlein. A large deviation principle in many-body quantum dynamics. Preprint arXiv:2010.13754. To appear in *Ann. Henri Poincaré.*

Bogoliubov theory in the Gross-Pitaevskii regime

In my third class, I discussed Bogoliubov theory in the Gross-Pitaevskii regime. My talk was based on the series of works:

C. Boccato, C. Brennecke, S. Cenatiempo, B. Schlein. Complete Bose-Einstein condensation in the Gross-Pitaevskii regime. *Comm. Math. Phys.* **359** (2018), no. 3, 975–1026. (Proof of BEC, for small potentials).

C. Boccato, C. Brennecke, S. Cenatiempo, B. Schlein. Optimal Rate for Bose-Einstein Condensation in the Gross-Pitaevskii Regime. *Comm. Math. Phys.* **376** (2020), no. 2, 1311–1395. (Proof of BEC, no smallness assumption).

C. Boccato, C. Brennecke, S. Cenatiempo, B. Schlein. Bogoliubov Theory in the Gross-Pitaevskii limit. *Acta Math.* **222** (2019), no. 2, 219–335. (Low-energy spectrum in the GP regime).

A "pedagogical" introduction to the approach I discussed can be found here:

B. Schlein. Bogoliubov excitation spectrum for Bose-Einstein condensates. Proc. Int. Cong. of Math. - 2018. Rio de Janeiro, Vol. 2 (2655-2672). Preprint arXiv:1802.06662.

Some more recent results on BEC:

S. Fournais. Length scales for BEC in the dilute Bose gas. Preprint arXiv:2011.00309. (Proof of BEC, beyond the GP regime).

C. Hainzl. Another proof of BEC in the GP-limit. Preprint arXiv:2011.09450. (Simpler proof of BEC, small potentials).

P. T. Nam, M. Napiórkowski, J. Ricaud, A. Triay. Optimal rate of condensation for trapped bosons in the Gross–Pitaevskii regime. Preprint arXiv:2001.04364. (Simpler proof of BEC for small potentials, trapping fields).

C. Brennecke, B. Schlein, S. Schraven. Bose-Einstein Condensation with Optimal Rate for Trapped Bosons in the Gross-Pitaevskii Regime. Preprint: arXiv:2102.11052. (BEC with trapping fields).

Correlation energy of mean-field fermions

In my last class, I presented results about correlation energy of Fermi gases in a linked mean-field and semiclassical limit. This talk was based on the two recent papers:

N. Benedikter, P. T. Nam, M. Porta, B. Schlein, and R. Seiringer. Optimal Upper Bound for the Correlation Energy of a Fermi Gas in the Mean-Field Regime. *Comm. Math. Phys.* **374** (2019), 2097–2150.

N. Benedikter, P. T. Nam, M. Porta, B. Schlein, and R. Seiringer. Correlation Energy of a Weakly Interacting Fermi Gas. Preprint arXiv:2005.08933. To appear in *Invent. Math.*

A previous result, which gives the correct correlation energy to second order in perturbation theory was proven in:

C. Hainzl, M. Porta, and F. Rexze. On the CorrelationEnergy of Interacting Fermionic Systems in the Mean-Field Regime. *Comm. Math. Phys.* **374** (2020), 485–524.

The formula for the correlation energy is consistent with prediction found in the physics literature. See for example:

M. Gell-Mann and K. A. Brueckner. Correlation energy of an electron gas at high density. *Phys. Rev.* **106** (1957), no. 2, 364–368.

More physics references can be found in the papers listed above.