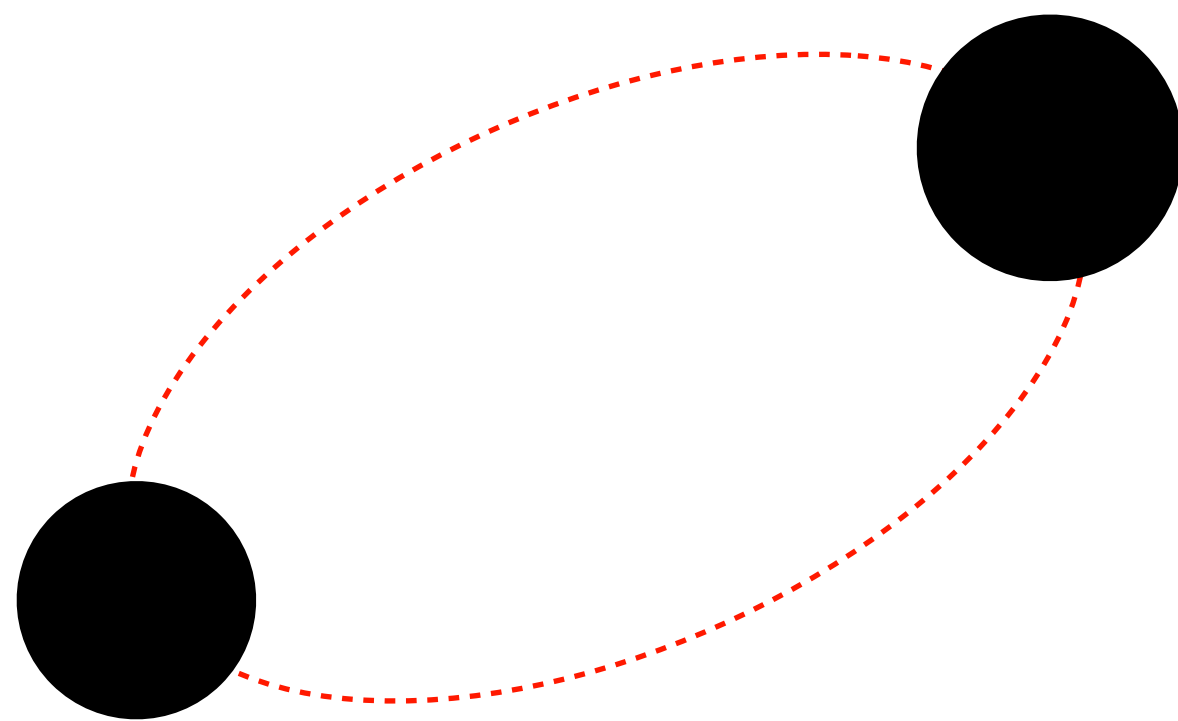


RINGDOWN OF PLUNGING BINARIES

LAURA PEZZELLA
SUPERVISOR: PROF. ANDREA MASELLI

BLACK HOLE BINARIES

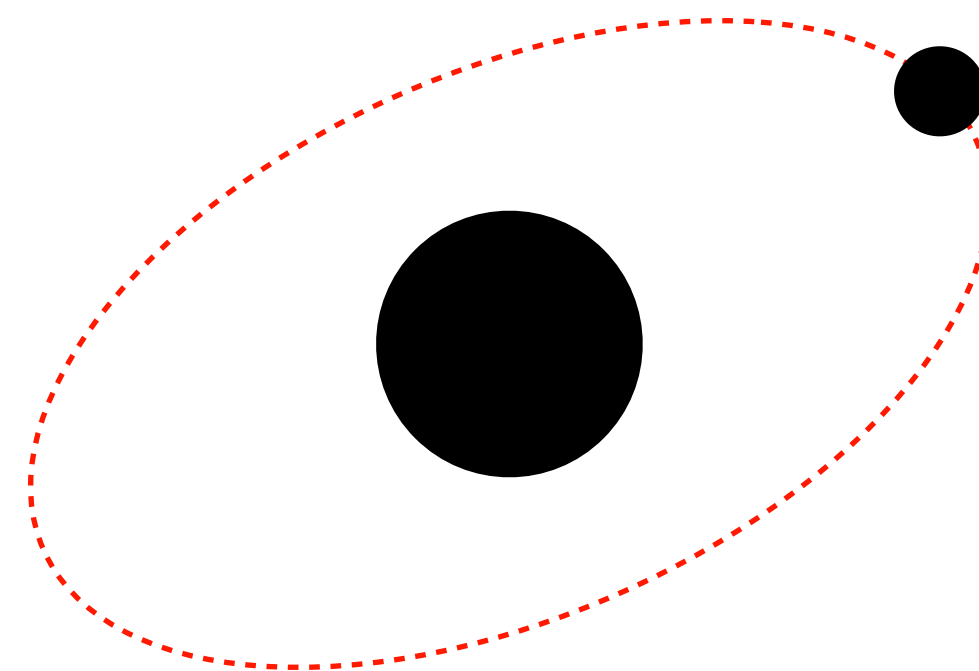
THREE REGIMES



Comparable masses

$$q = m_2/m_1 \sim 1$$

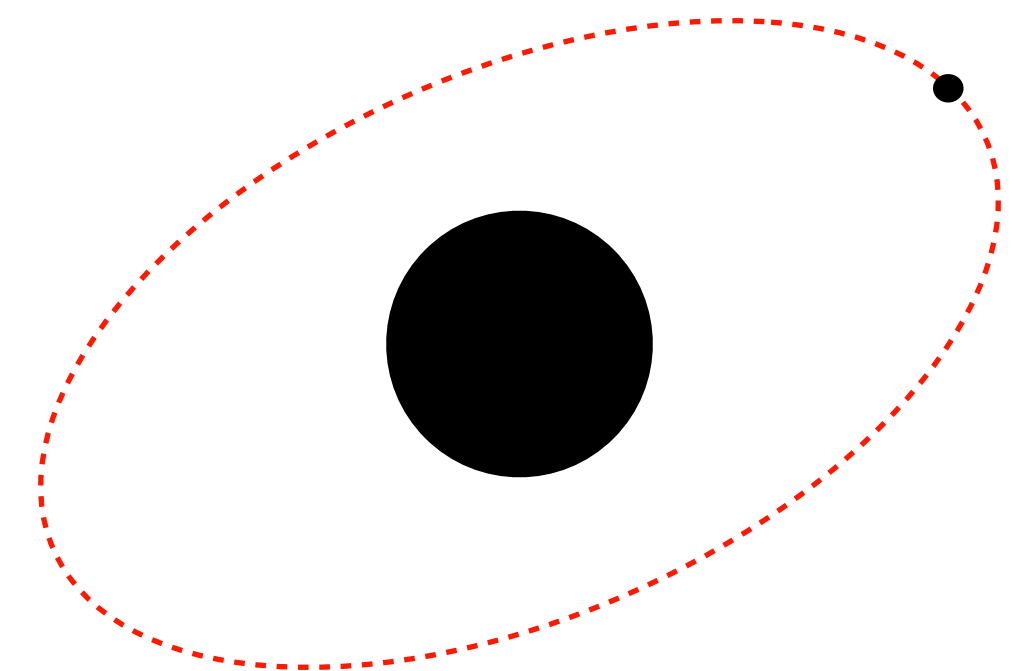
LVK - ET/CE



Intermediate Mass Ratio

$$q = m_2/m_1 \sim 10^{-1} - 10^{-3}$$

ET/CE - LGWA - LISA



Extreme Mass Ratio

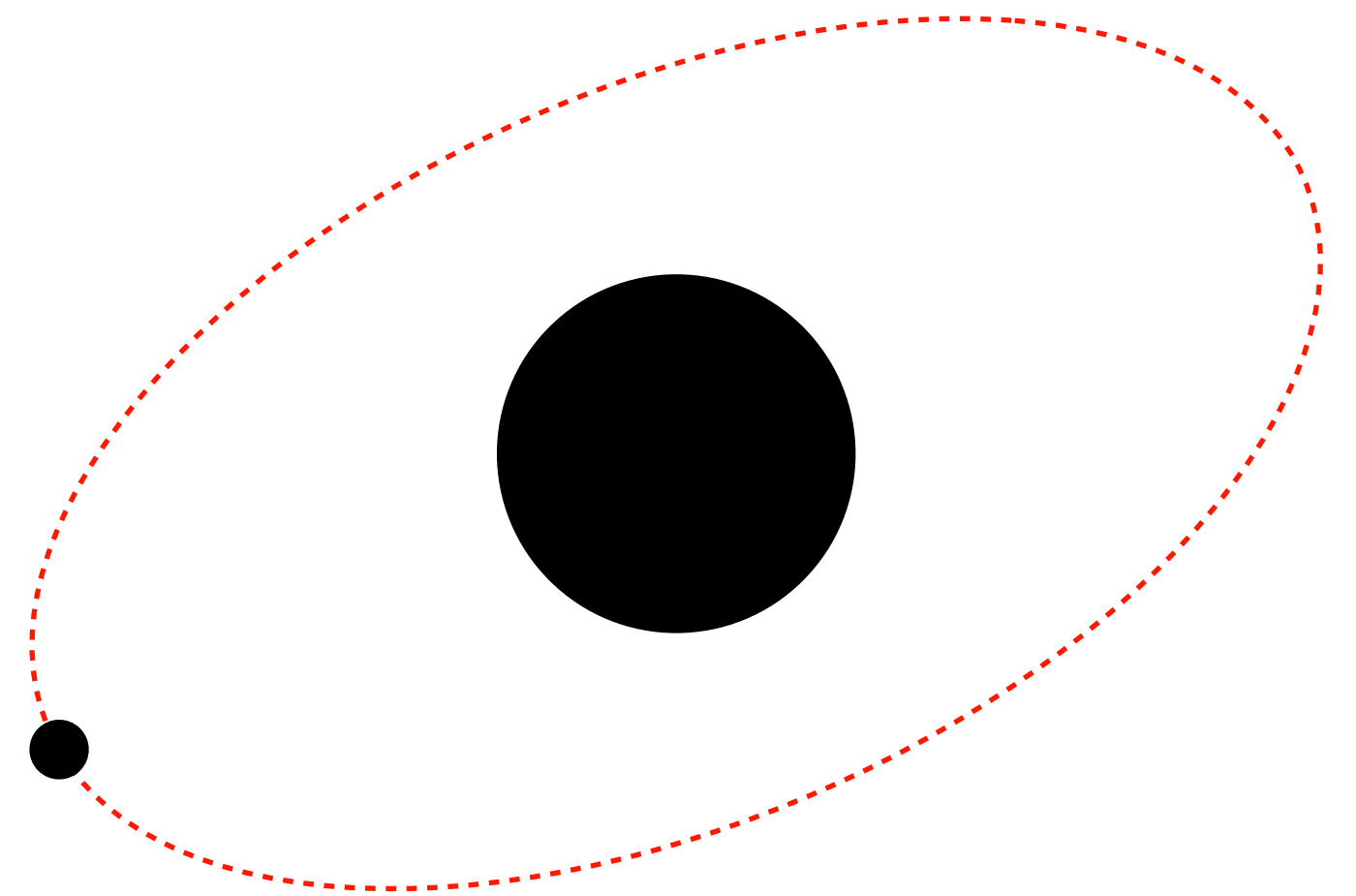
$$q = m_2/m_1 \lesssim 10^{-4}$$

LISA

EVOLUTION OF PLUNGING BINARIES

THREE STAGES

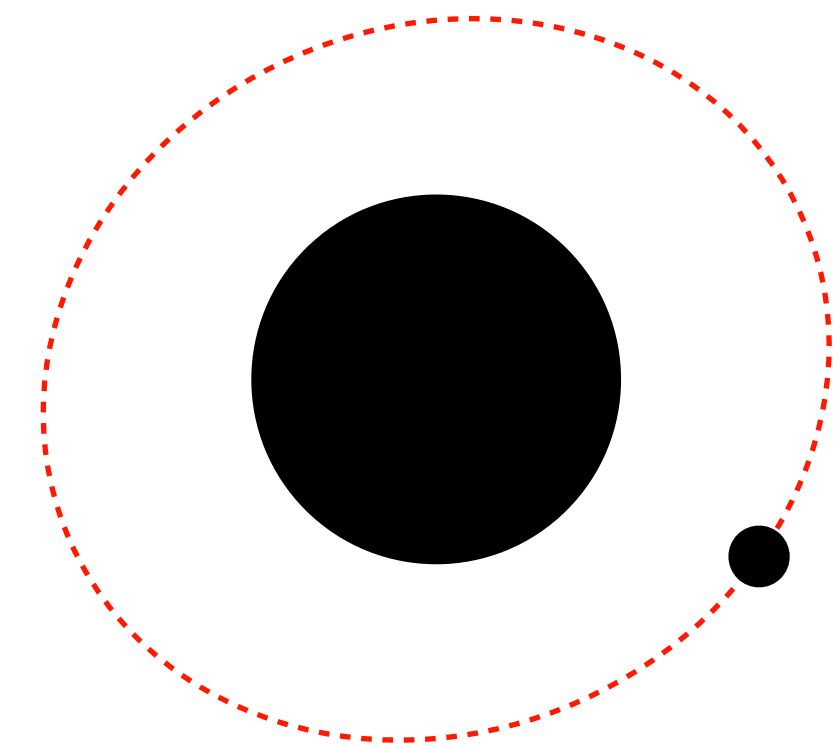
- **Inspiral regime:** the orbit of the secondary body evolves adiabatically. Extreme mass ratio inspiral simulations end to the ISCO;



EVOLUTION OF PLUNGING BINARIES

THREE STAGES

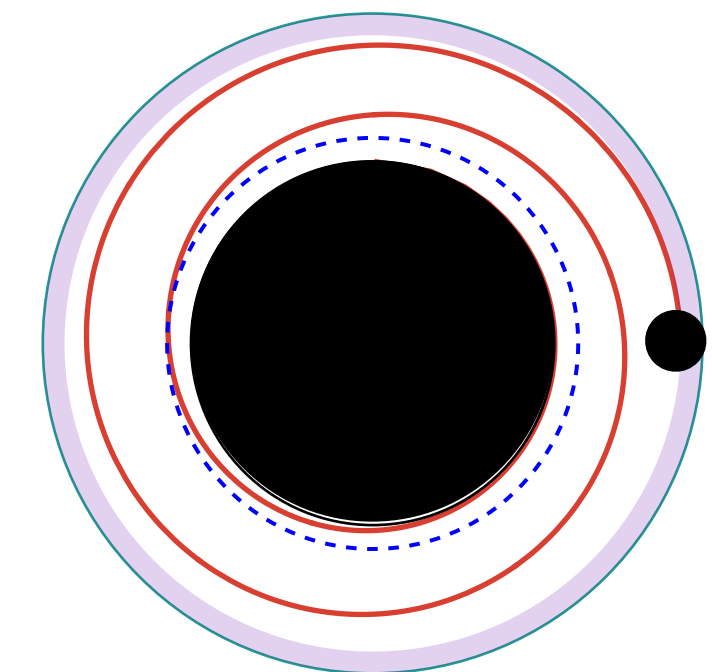
- **Inspiral regime:** the orbit of the secondary body evolves adiabatically. Extreme mass ratio inspiral simulations end to the ISCO;
- **Transition regime:** the character of the orbit gradually changes as the secondary approaches the **ISCO** of the primary BH;



EVOLUTION OF PLUNGING BINARIES

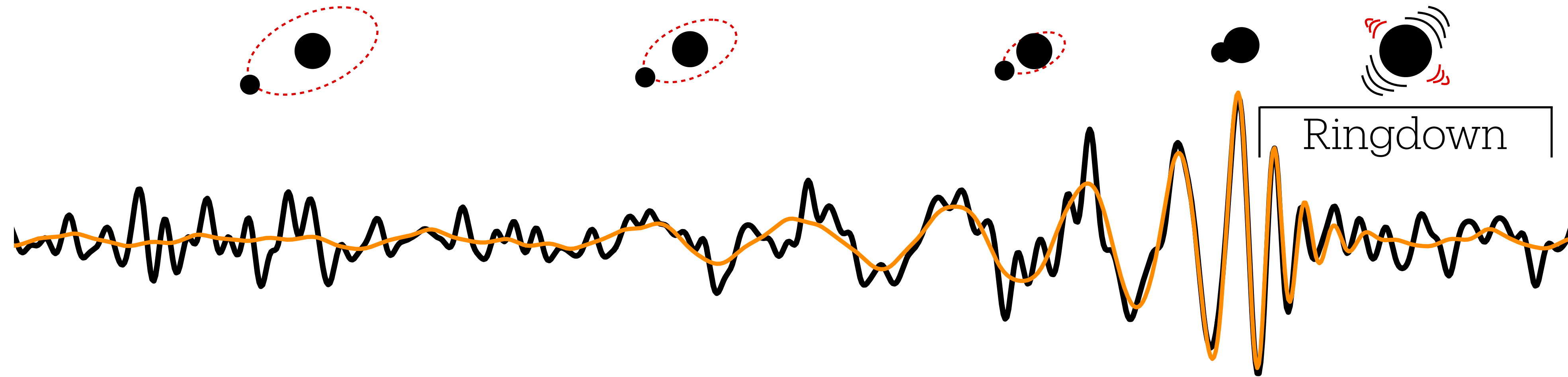
THREE STAGES

- **Inspiral regime:** the orbit of the secondary body evolves adiabatically. Extreme mass ratio inspiral simulations end to the ISCO;
- **Transition regime:** the character of the orbit gradually changes as the secondary approaches the **ISCO** of the primary BH;
- **Plunge:** the body plunges into the horizon along a geodesic.



RINGDOWN

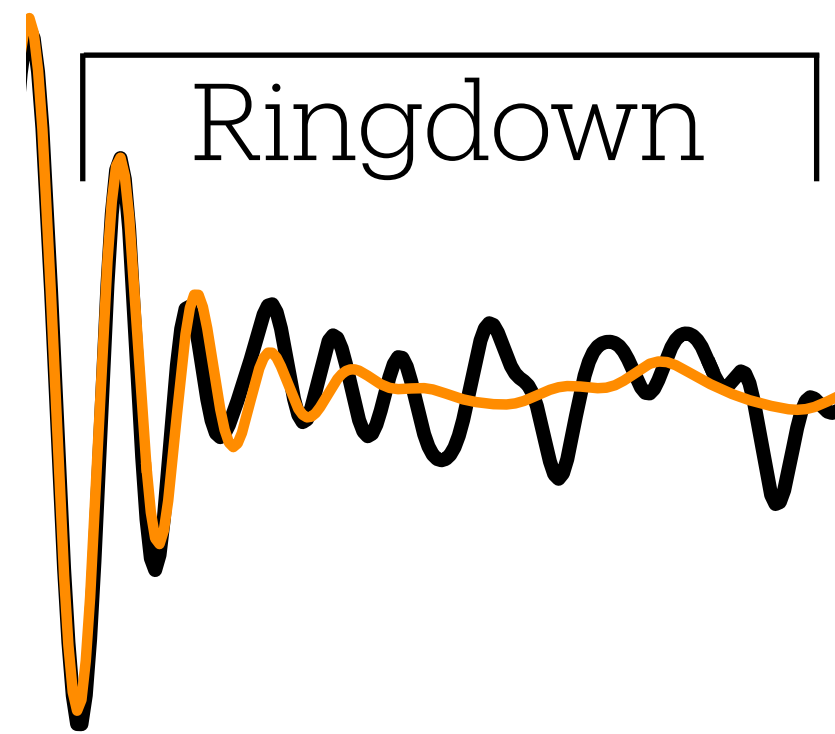
QUASI NORMAL MODES EXCITATION



- The ringdown waveform originates from the **distorted final product** of the merger.
- The **gravitational signal** emitted during the ringdown is well modeled by a **superposition** of damped sinusoids.
- The characteristic complex frequencies are called quasi-normal modes (**QNMs**)

RINGDOWN

QUASI NORMAL MODES EXCITATION



$$h(t) = \sum_{\ell mn} A_{\ell mn} e^{-t/\tau_{\ell mn}} \cos(\omega_{\ell mn} t + \phi_{\ell mn})$$

QNM frequencies depend entirely on the **final BH's parameters** (mass **M** and spin **J** in vacuum GR)

The **amplitudes and phases** of the signal depend on the **dynamics** of the **specific process** that formed the BH

FRAMEWORK

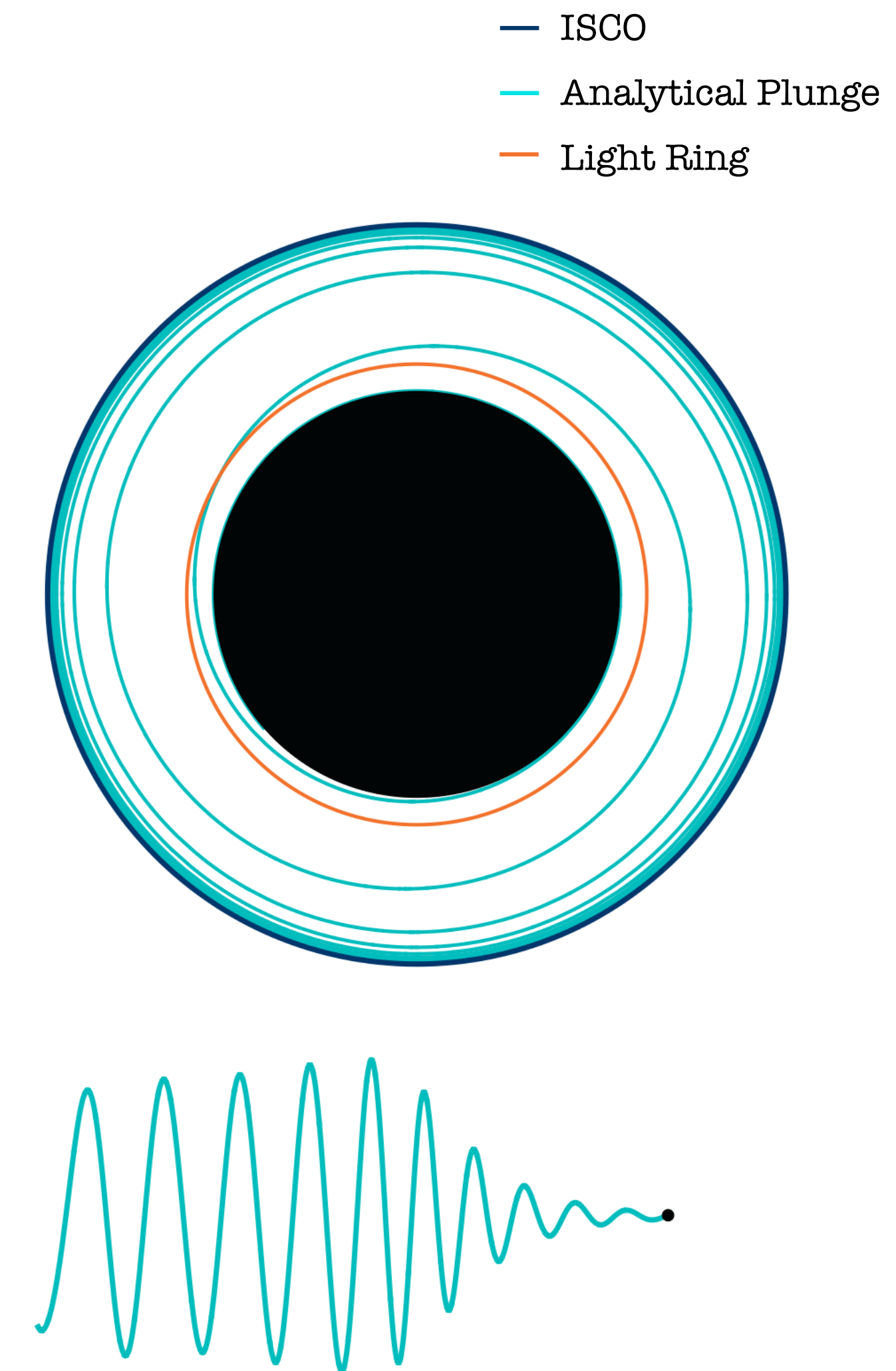
WHAT DO WE DO?

- **Kerr** BHs
- **Equatorial** orbits ($\theta = \pi/2$)
- **Circular orbit** near the ISCO
- Edge-on binary
 $(\theta_{\text{source}}, \phi_{\text{source}}) = (\pi/2, 0)$
- **Goal:** Compute the excitation coefficients for particles plunging from the innermost stable circular orbit into a Kerr BH
- **How:** Compute these quantities for extreme mass-ratio binaries using **BH perturbation theory**

PLUNGING GEODESICS

DYSON-VAN DE MEERT

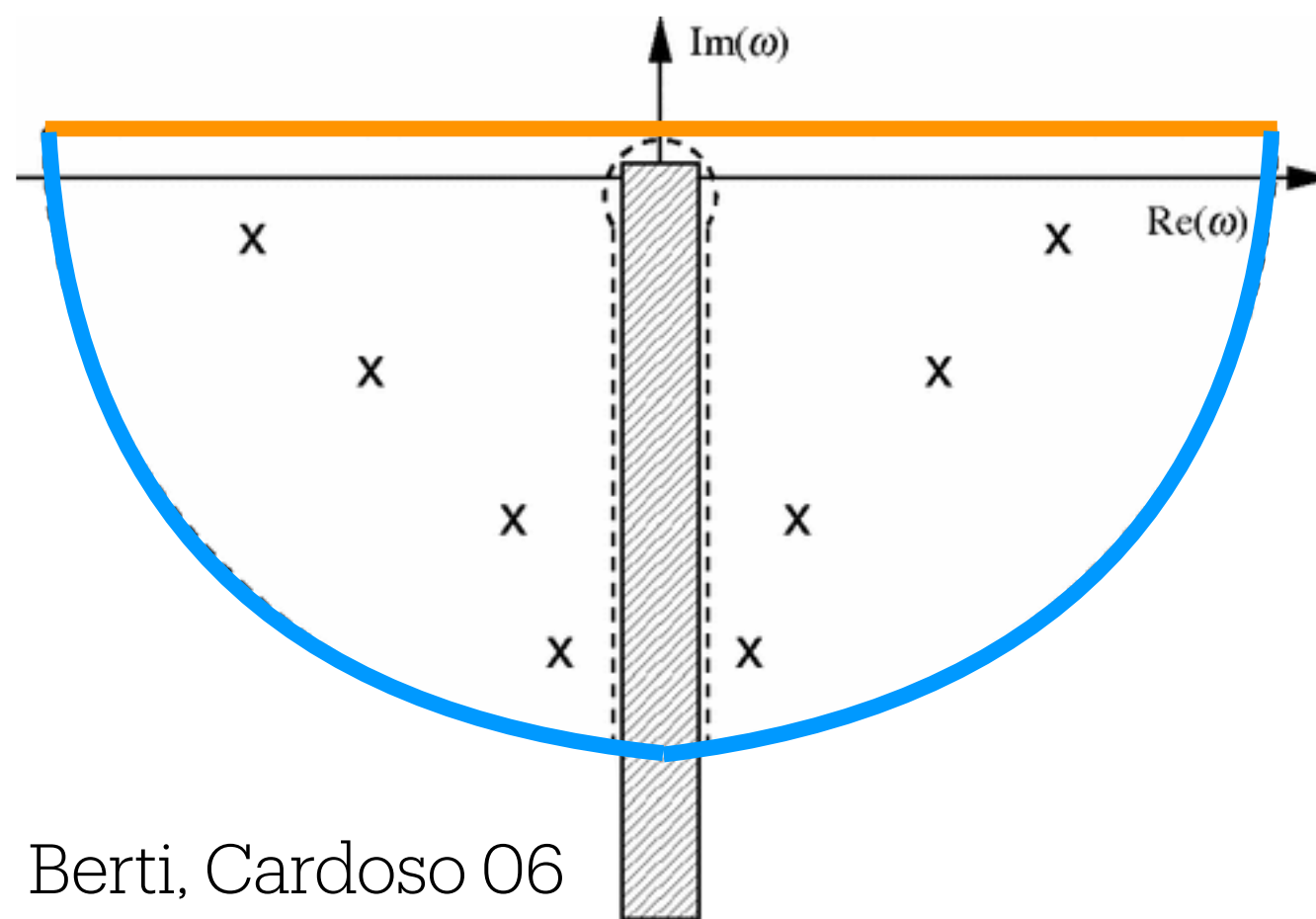
- **Analytical expression** for geodesics plunging found by Dyson and van de Meent in terms of elliptic functions.
- In this model, the plunge is **analytically extended up to the ISCO**: the transition is not modelled.
- The body is in **free fall**, with energy and angular momentum fixed at the ISCO values.



RESIDUAL THEOREM

QNMS AS POLES

- QNMs are the **poles** of the Green Function
- Integration along the real axis is replaced by the sum of residuals
- Applying the residual theorem, the **time domain** solution reads



Berti, Cardoso 06

$$\begin{aligned}\hat{\tilde{X}}_p^{\ell m}(u, v \rightarrow \infty) &\approx - \sum_n 2\pi i \operatorname{Res}_{\omega \rightarrow \omega_{\ell mn}} \int_{\mathbb{R}} d\omega e^{-i\omega t} \tilde{X}_p^{\ell m \omega}(r \rightarrow +\infty) \\ &= -2\pi \sum_n C_{\ell mn} e^{-i\omega_{\ell mn} u}\end{aligned}$$

EXCITATION COEFFICIENTS

QUASI NORMAL MODES EXCITATION

$$\hat{\tilde{X}}_p^{\ell m}(u, v \rightarrow \infty) = -2\pi \sum_n C_{\ell mn} e^{-i\omega_{\ell mn} u}$$

- The quasinormal **excitation coefficients** are a concrete measure of the QNM content of a waveform
- Excitation coefficients can be written as the product of two contributions: the quasinormal **excitation factors**, which are **initial-data independent**, and the **source-dependent integral** $I_{\ell mn}$

$$C_{\ell mn} = B_{\ell mn} I_{\ell mn} \quad B_{\ell mn} = \frac{\tilde{A}_{\ell mn}}{2\omega_{\ell mn} \alpha_{\ell mn}}$$

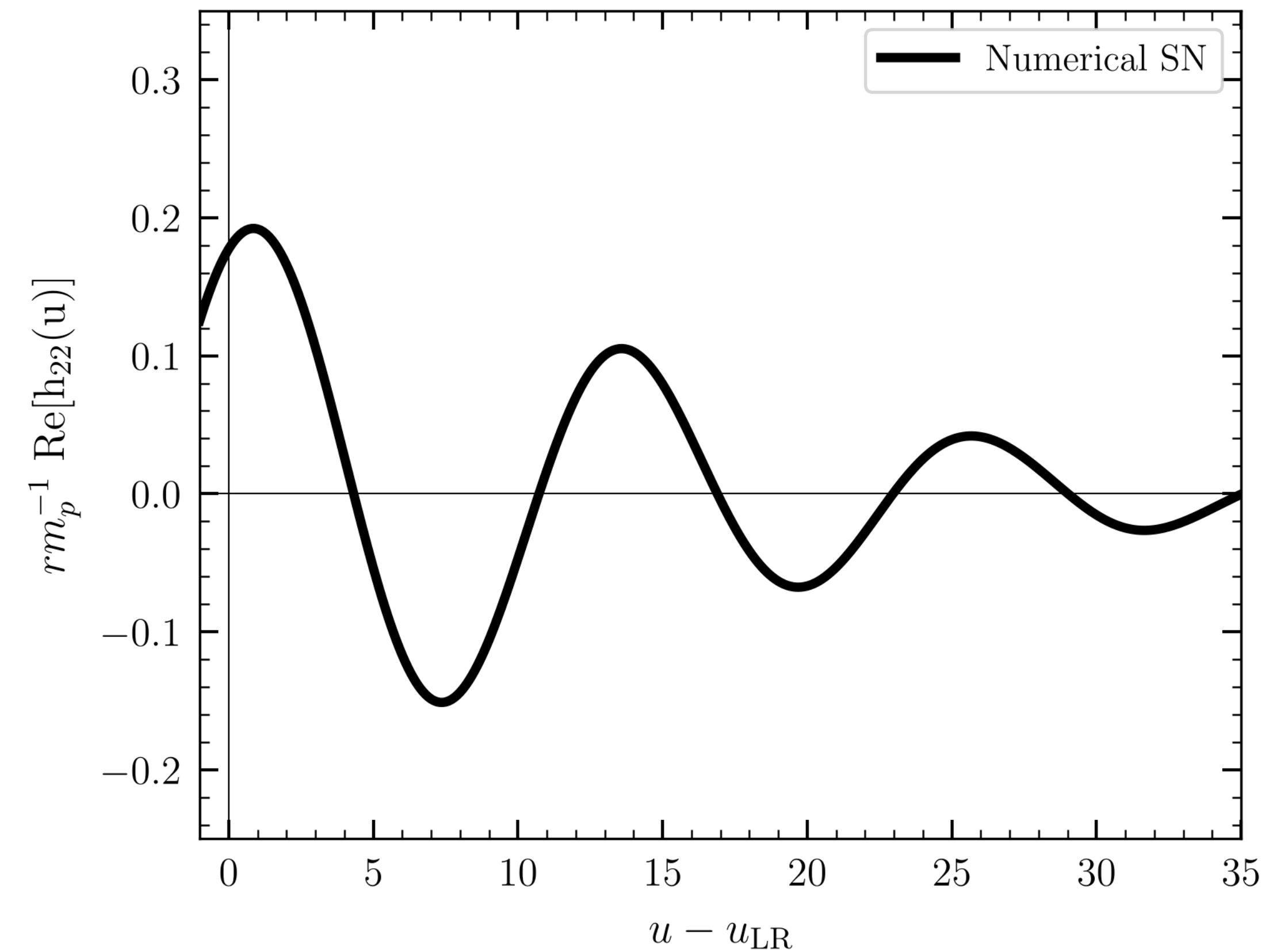
RINGDOWN

QUASI NORMAL MODES EXCITATION

M. Della Rocca, LP + (2026)

The **amplitudes** of the signal can be rewritten in terms of the **excitation coefficients**.

$$\begin{aligned} h_{\ell m} &= \frac{8}{\sqrt{2\pi}} \frac{1}{r} \int d\omega e^{-i\omega t} {}_{-2}S_{\ell m}^{a\omega}(\theta, \phi) \frac{\tilde{X}_{\ell m \omega}}{\sqrt{c_0}} \\ &= 8 \frac{1}{r} \sum_n C_{\ell mn} e^{-i\omega_{\ell mn} u} {}_{-2}S_{\ell m}^{a\omega}(\theta, \phi) \end{aligned}$$

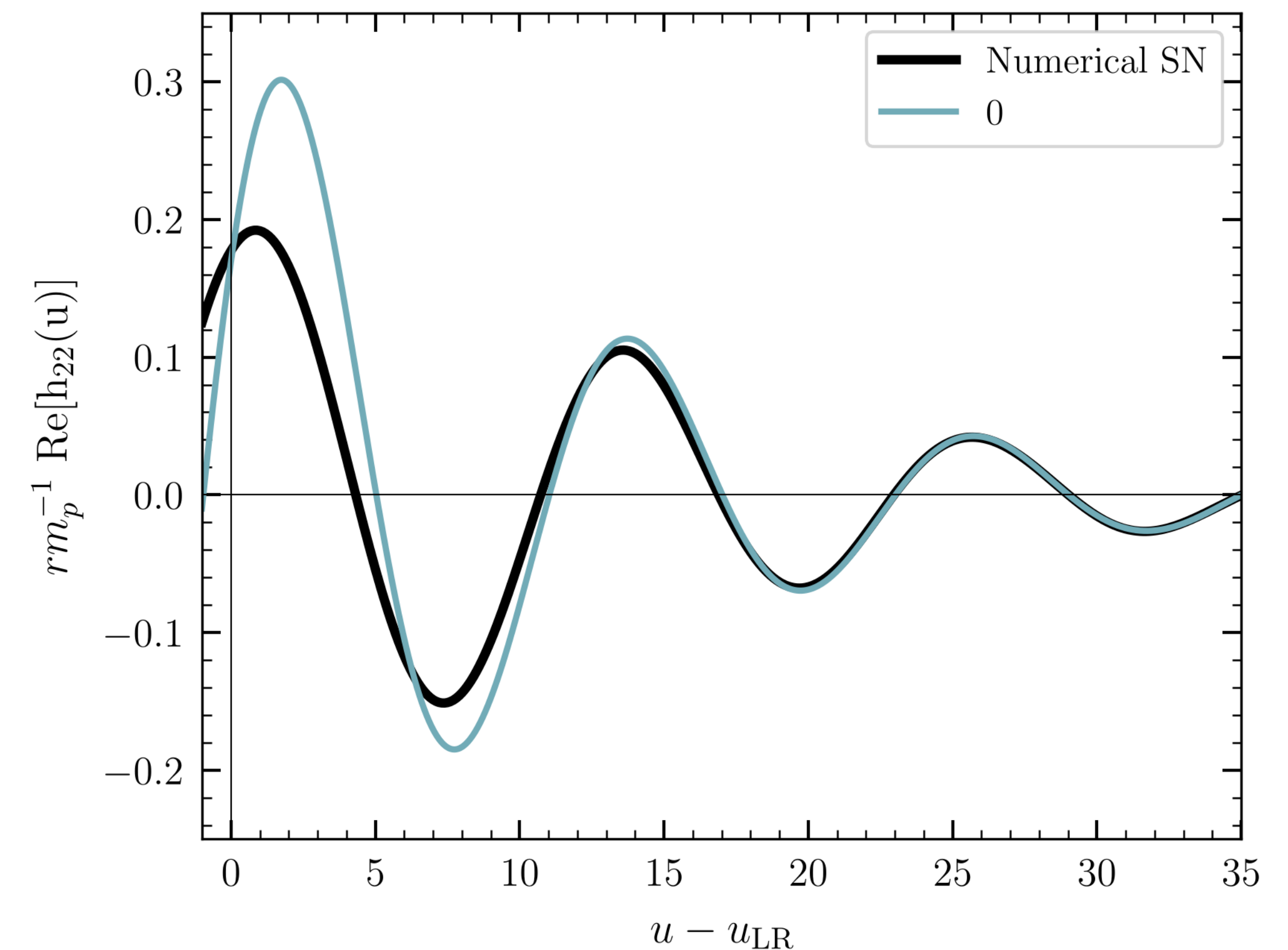


GRAVITATIONAL WAVE EMISSION

COMPARISON WITH NUMERICAL SOLUTIONS

M. Della Rocca, LP + (2026)

By adding higher overtones improves the agreement with the numerical waveforms.



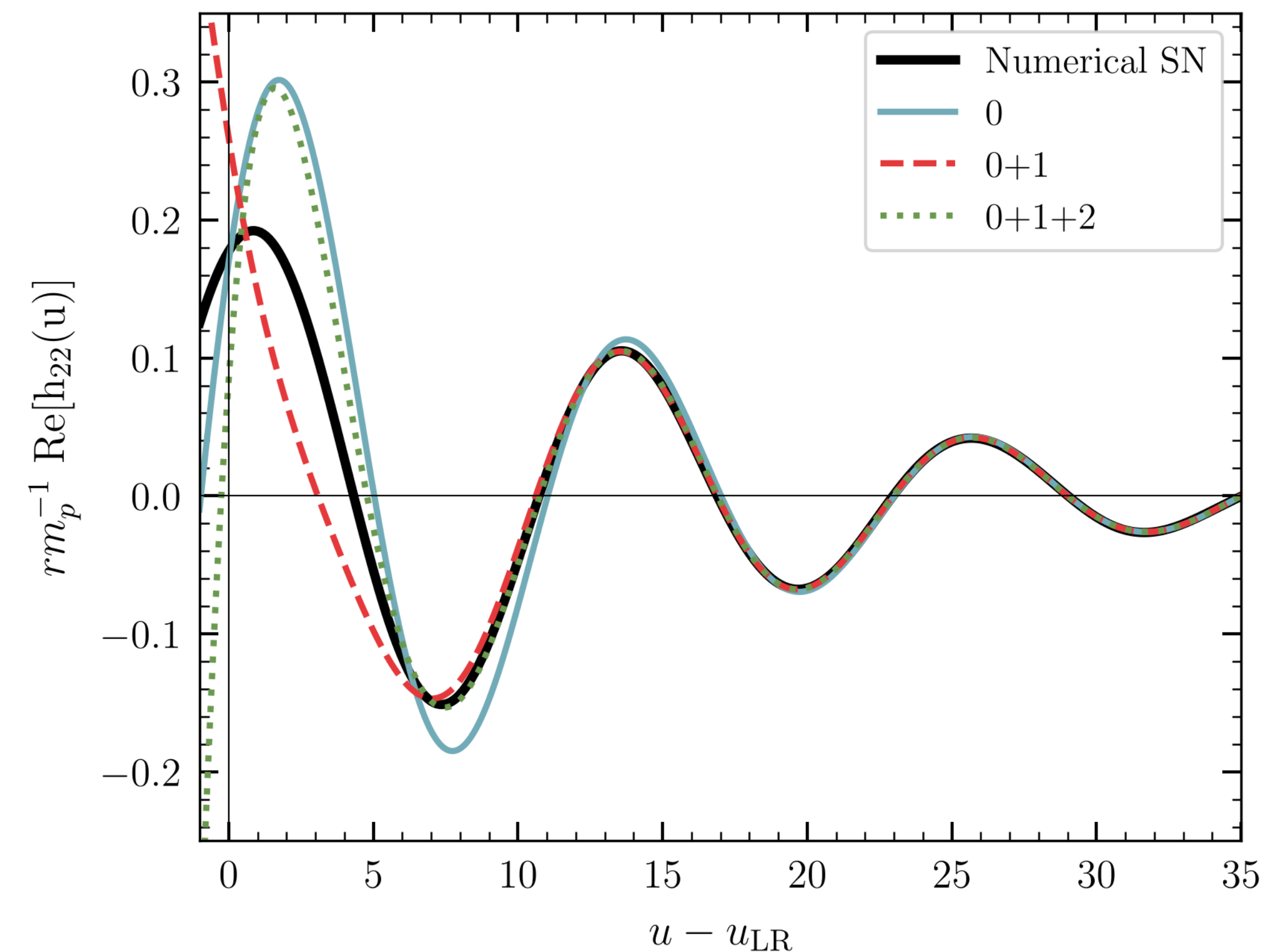
GRAVITATIONAL WAVE EMISSION

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The more the modes, the **smaller the discrepancy**.



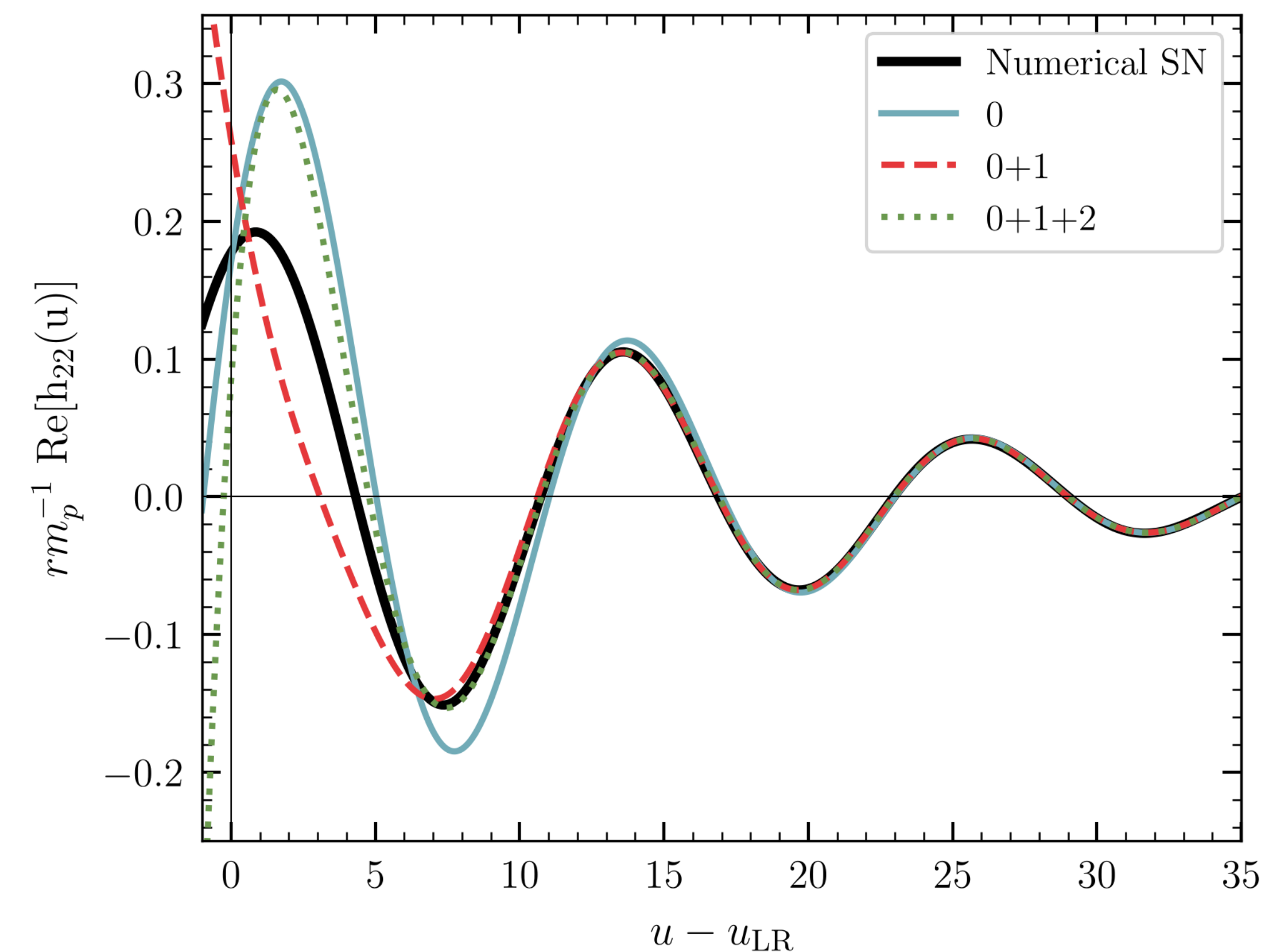
GRAVITATIONAL WAVE EMISSION

COMPARISON WITH NUMERICAL SOLUTIONS

M. Della Rocca, LP + (2026)

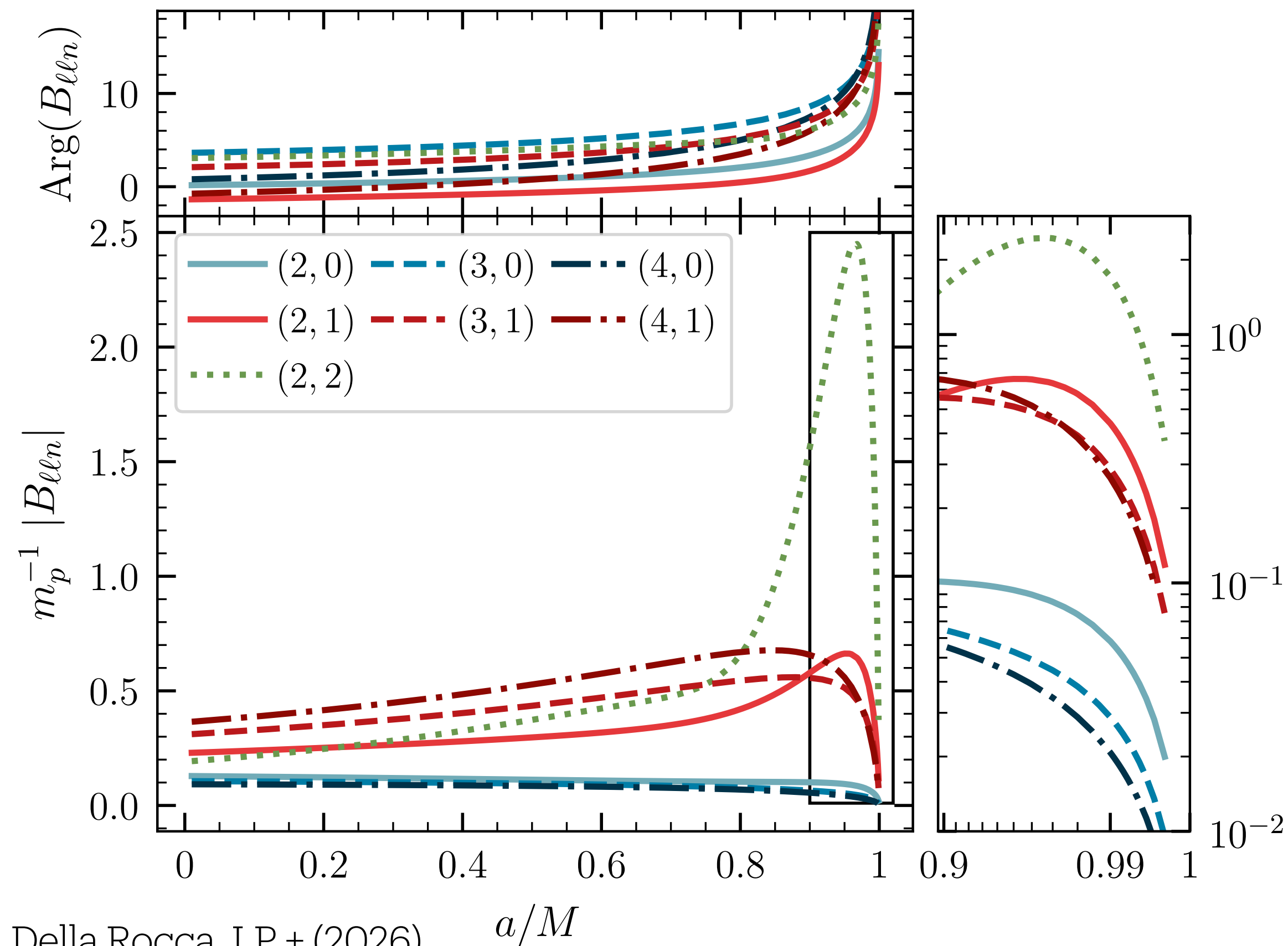
By adding higher overtones improves the agreement with the numerical waveforms.

The more the modes, the **smaller the discrepancy**.



EXCITATION FACTORS

AT DIFFERENT SPINS (ℓ, ℓ, n)

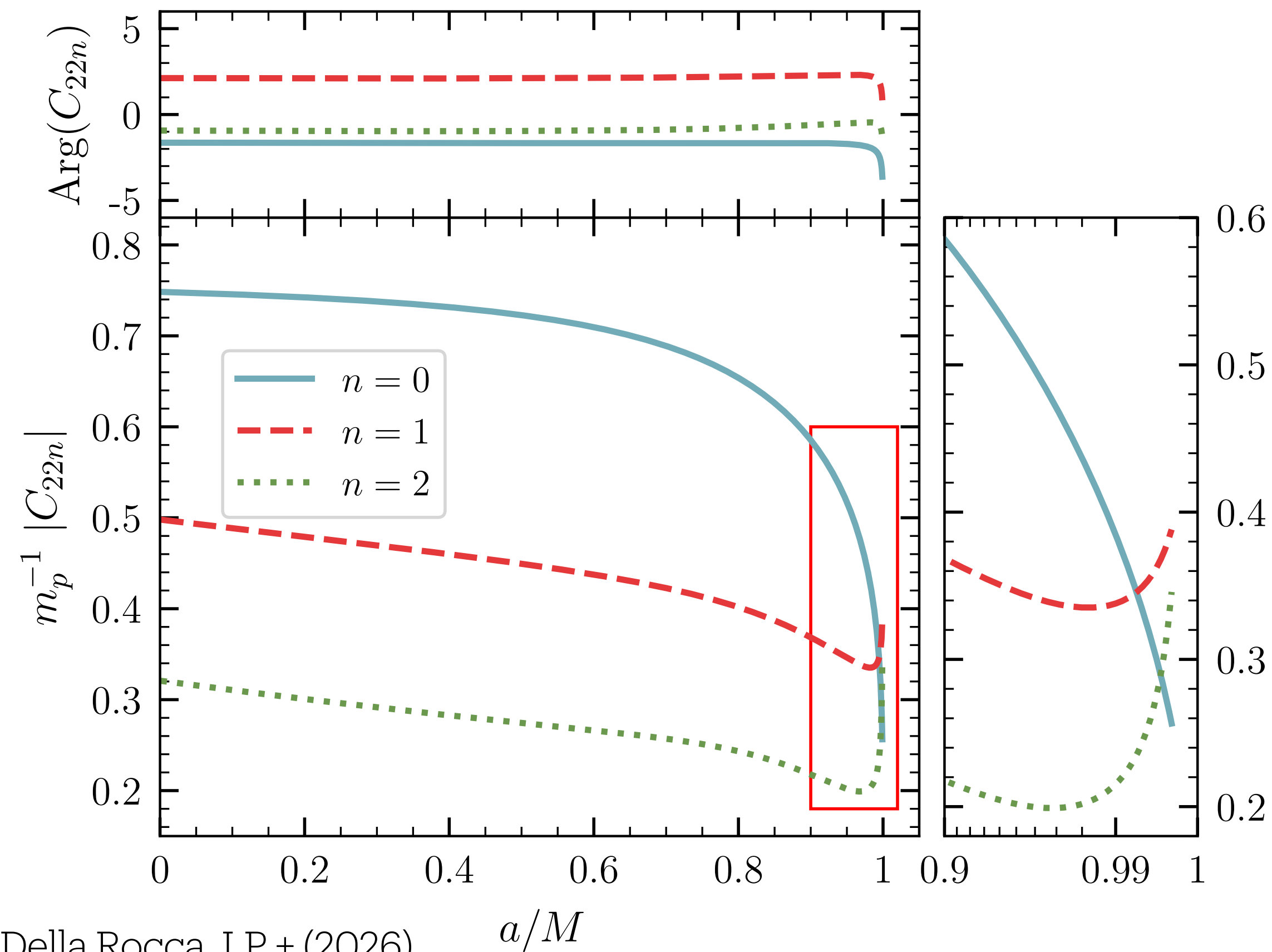


- The excitation factors are **source independent**;
- Different behaviour at high spins;

M. Della Rocca, LP + (2026) a/M

EXCITATION COEFFICIENTS

AT DIFFERENT SPINS (2,2)

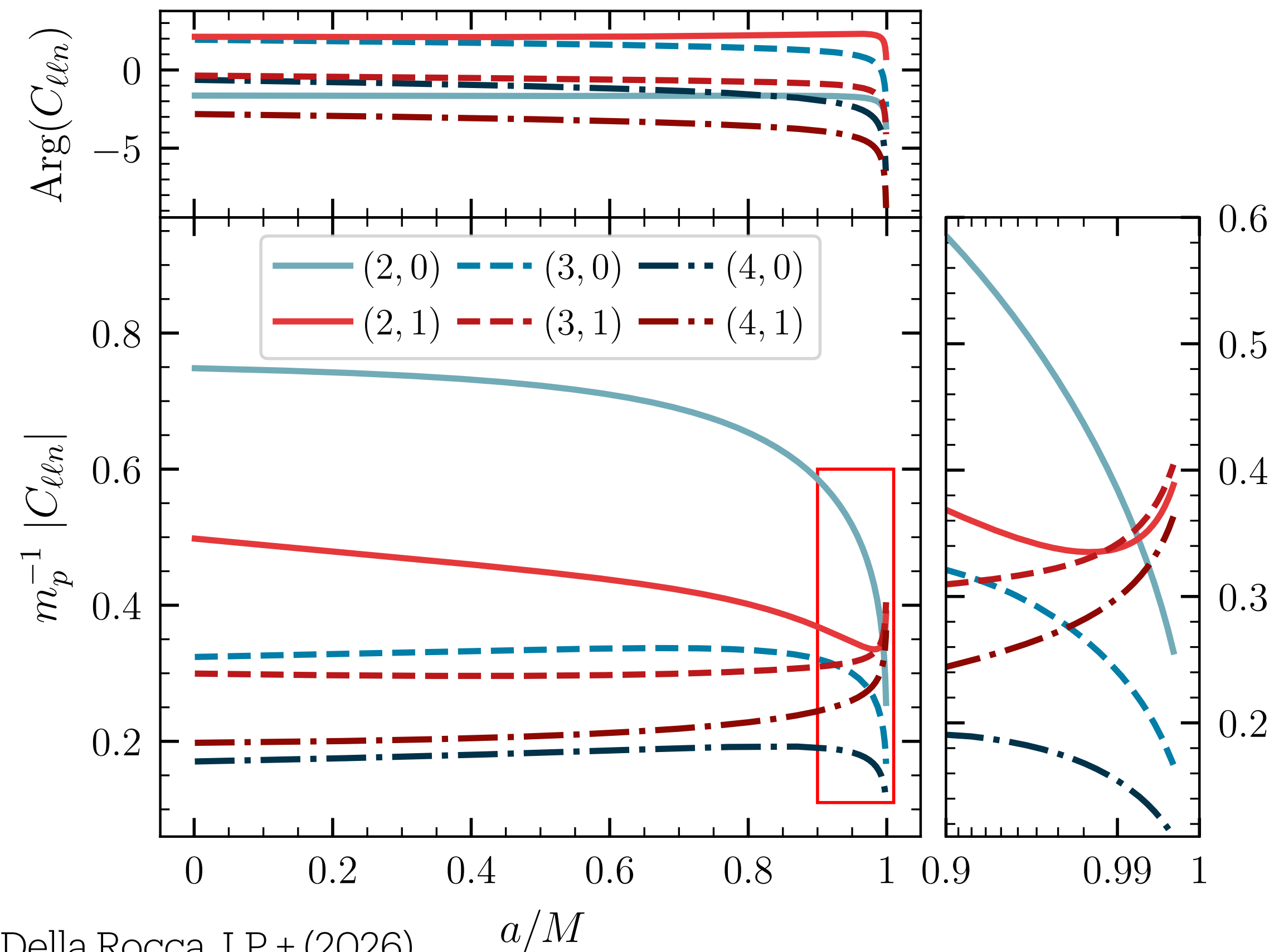


M. Della Rocca, LP + (2026)

- Computation of the excitation coefficient C_{22n} for **different spins**;
- Different trends increasing the **overtone number**;
- Overtones may play a dominant role for **near-extremal Kerr BHs**.

EXCITATION COEFFICIENTS

AT DIFFERENT SPINS (ℓ, ℓ, n)

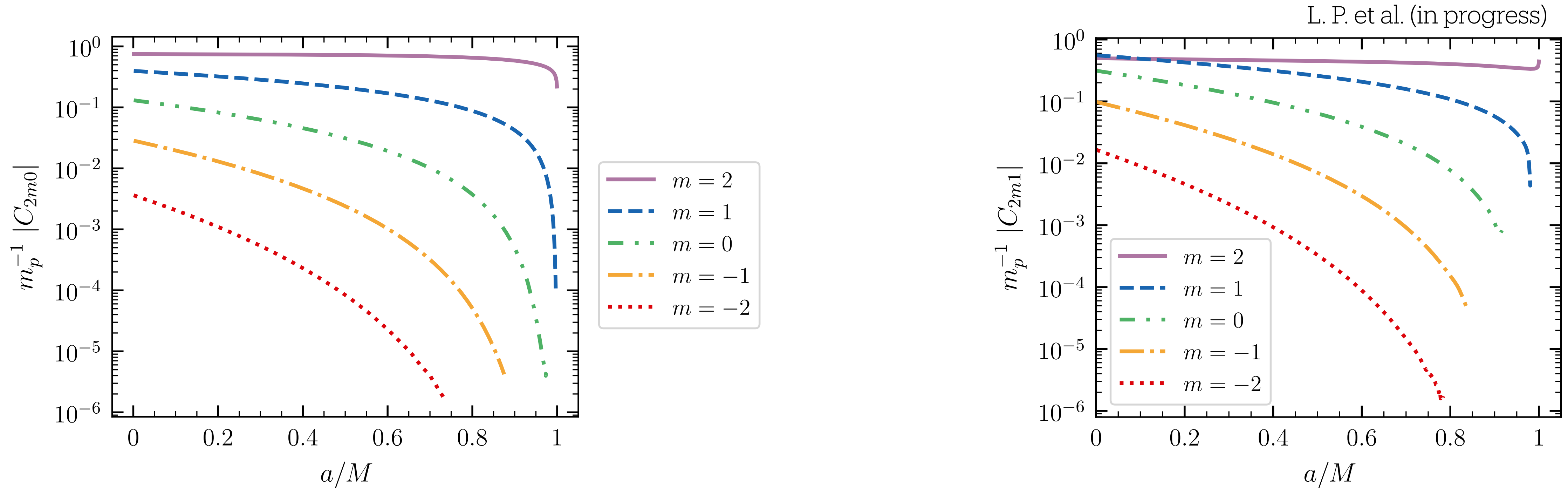


M. Della Rocca, LP + (2026) a/M

- Computation of the excitation coefficient $C_{\ell\ell 0}$ and $C_{\ell\ell 1}$ for different spins
- For nearly extremal BH, the **overtone contribution** becomes more **relevant** at any ℓ

EXCITATION COEFFICIENT

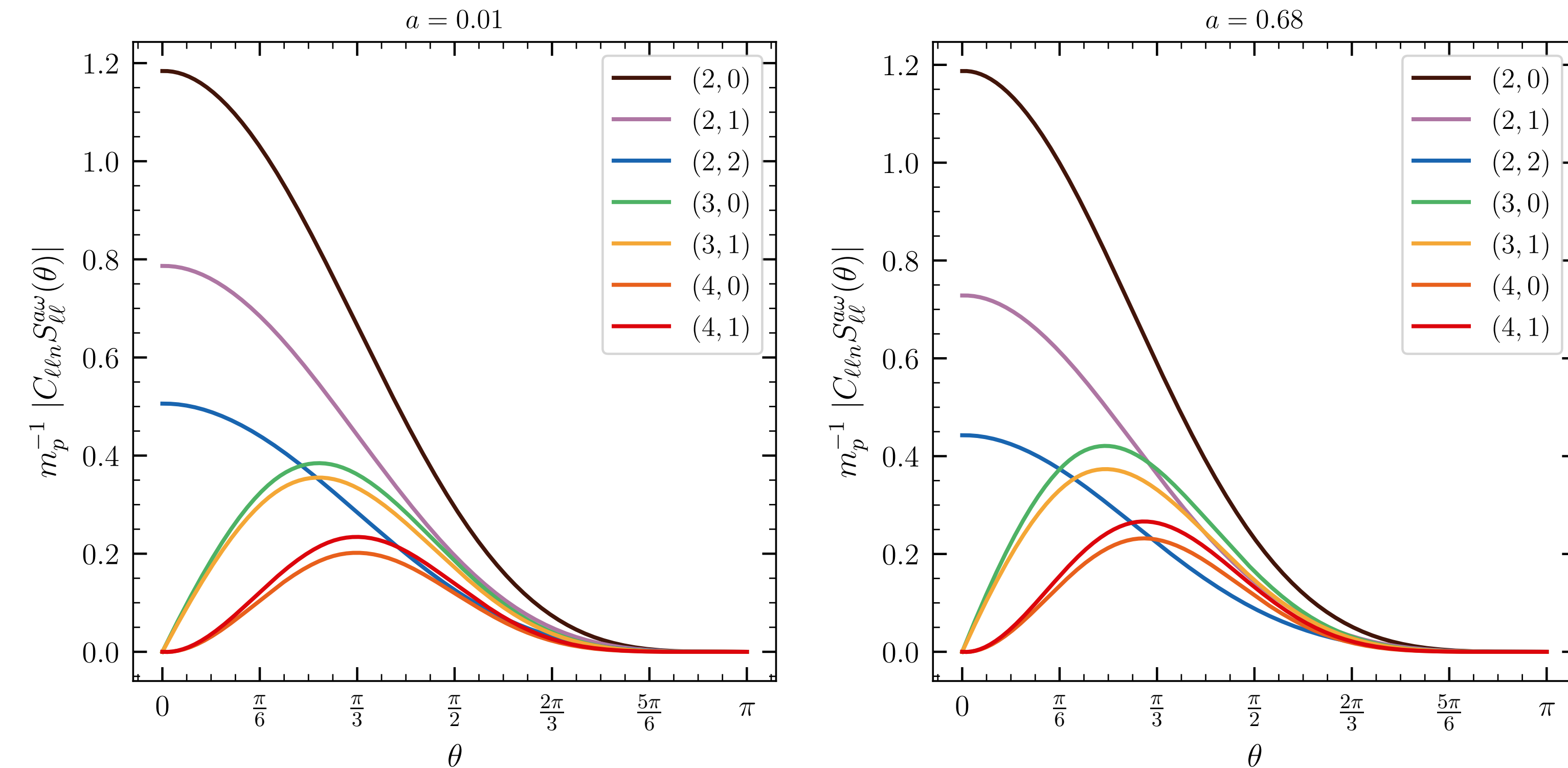
THE CONTRIBUTION OF THE $\ell \neq m$ MODES



- Computation of the excitation coefficient $C_{\ell mn}$ for different spins;
- New tables for $\ell = \{2,3,4\}$ and $n = \{0,1\}$

INCLINATION OF THE SOURCE

WHAT HAPPENS WHEN WHAT WE SEE IS NOT EDGE-ON?



L. P. et al. (in progress)

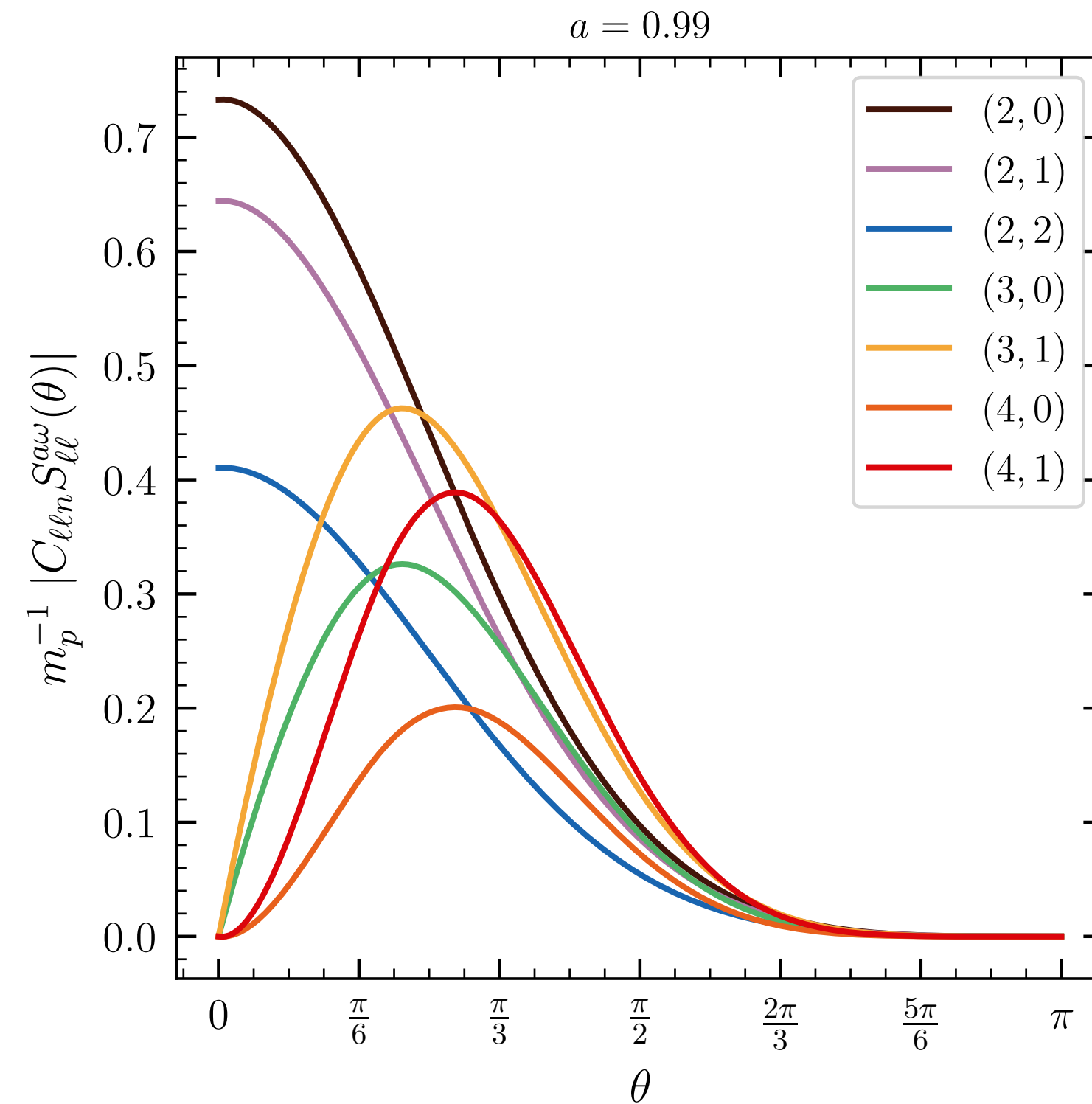
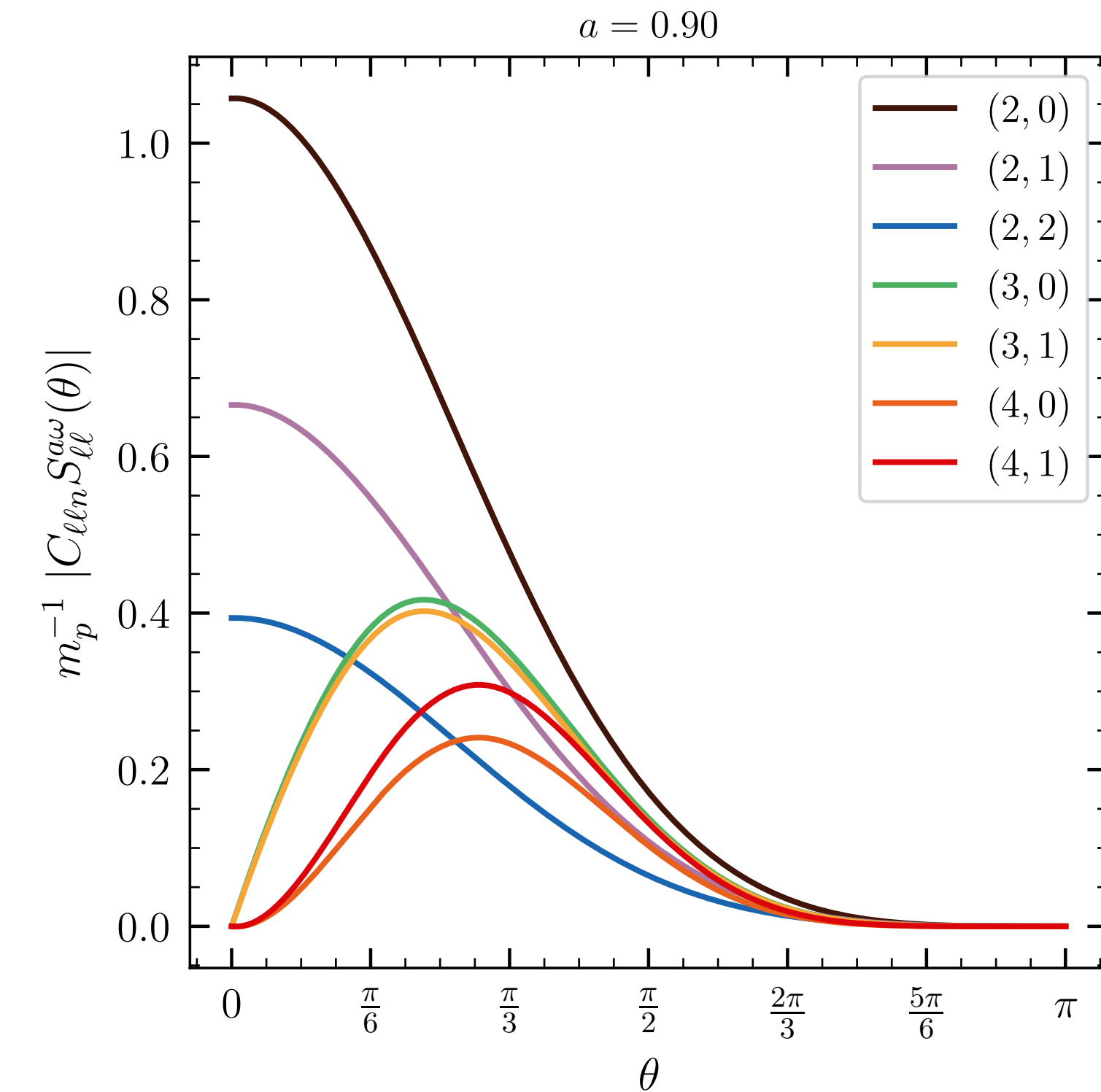
- Full waveform:

$$h_{\ell m} = 8 \frac{1}{r} \sum_n C_{\ell m n} e^{-i\omega_{\ell m n} u} {}_{-2}S_{\ell m}^{a\omega}(\theta, \phi)$$

- Observed waveform amplitude depends on the source position (θ, ϕ) .
- Different mode contribution for different polar angles

INCLINATION OF THE SOURCE

HIGH SPIN CASE



- Full waveform:

$$h_{\ell m} = 8 \frac{1}{r} \sum_n C_{\ell m n} e^{-i\omega_{\ell m n} u} {}_{-2}S_{\ell m}^{a\omega}(\theta, \phi)$$

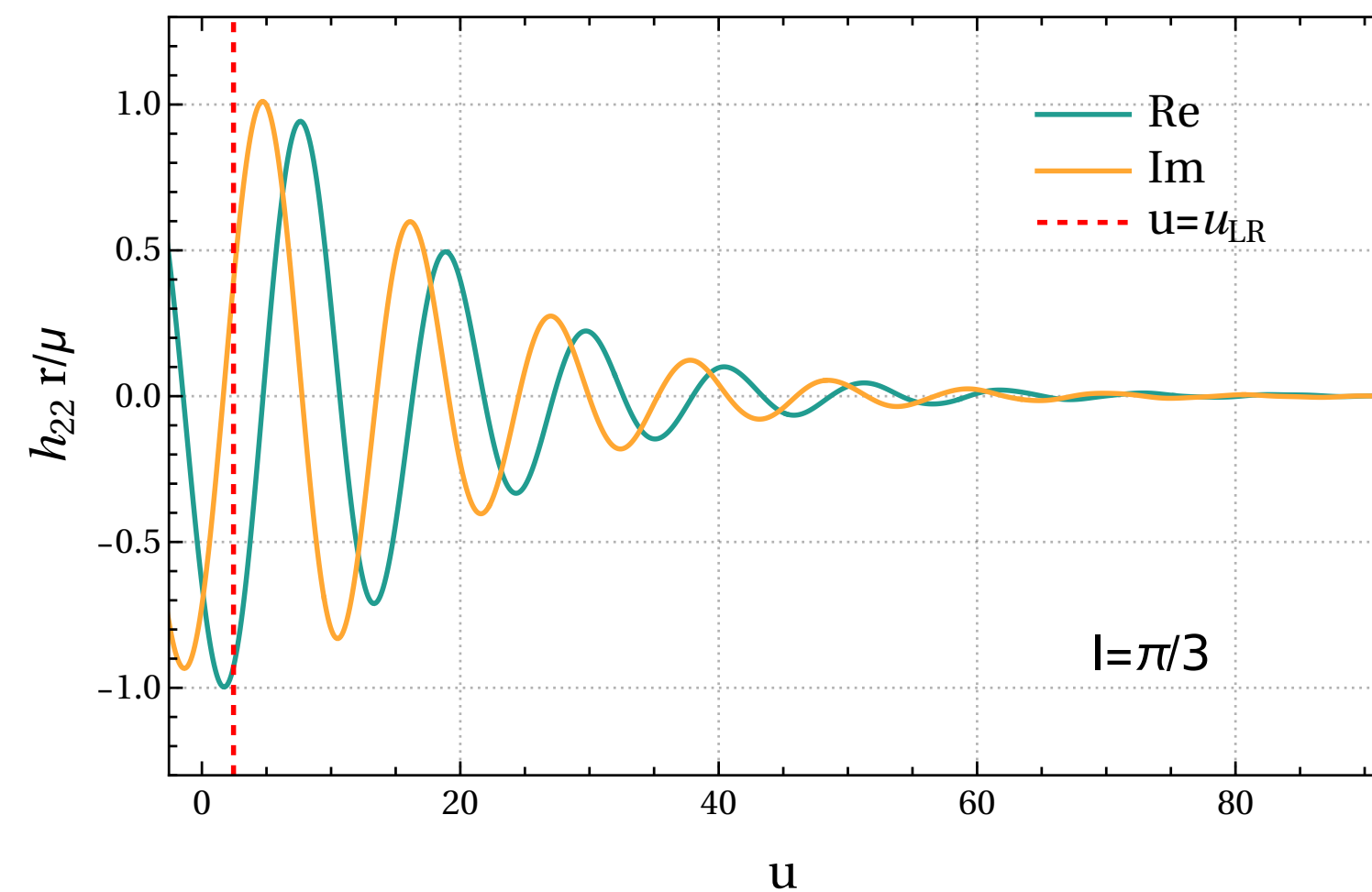
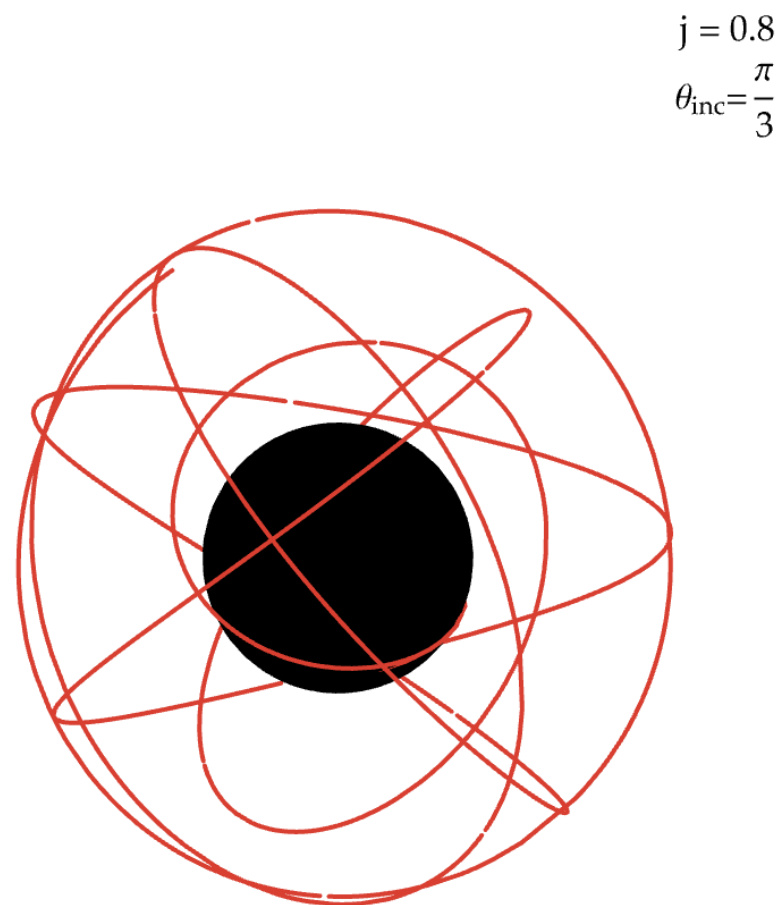
- Observed waveform amplitude depends on the source position (θ, ϕ) .
- Different mode contribution for different polar angles

L. P. et al. (in progress)

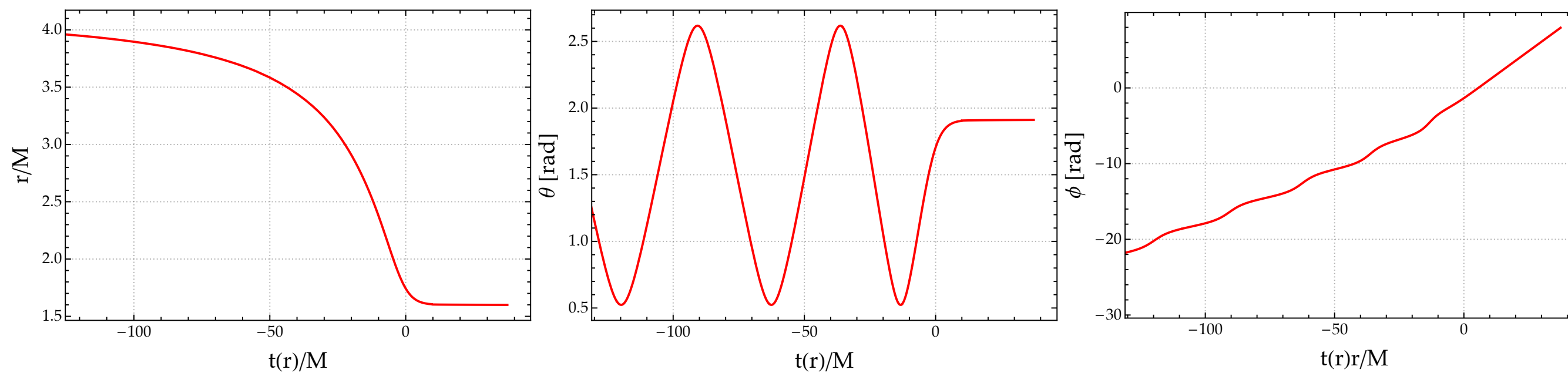
INCLINED PLUNGE

WHAT HAPPENS WHEN THE PLUNGE IS NOT EQUATORIAL?

L. P. et al. (in progress)



- The inclination is characterised by the inclination angle I ;
- For non-equatorial orbits, $Q \neq 0$ and $90^\circ - I \leq \theta \leq 90^\circ + I$ (prograde orbits);
- The source term has now an additional contribution.



CONCLUSIONS

RESULTS AND PROSPECTS

- * Developed a code to **treat the plunge** of the secondary in **frequency domain**;
- * The **complete waveform** can be reconstructed as the **superposition of excitation coefficient**, provided that more modes are included;
- * New catalogue of **excitation coefficients** C_q for plunging events;

What we are working on:

- ❑ Expand the catalogue for $\ell \neq m$ and characterise the mode contribution as function of the inclination of the source;
- ❑ Characterise the plunge as a function of **inclination** (and, possibly, **eccentricity**);
- ❑ Check the **validity** of the method for **comparable mass binaries**.

THANKS FOR THE ATTENTION!

LAURA PEZZELLA
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October 13th 2026

PUBLICATIONS

- [L. Pezzella](#), K. Destounis, A. Maselli, V. Cardoso, *Quasinormal modes of black holes embedded in halos of matter*, doi.org/10.1103/PhysRevD.111.064026
- M. Della Rocca, [L. Pezzella](#), E. Berti, L. Gualtieri, A. Maselli, *Quasinormal ringing of Kerr black holes III. Excitation factors by an equatorially plunging particle*, Phys. Rev. D 114, 044036, 11 August 2026, doi.org/10.1103/ydp5-ktpn
- J. Tissino, F. Santoliquido, F. Iacovelli, U. Dupletsa, T. Dal Canton, M. Ballelli, A. Chopra, L. E. Espinosa Castro, [L. Pezzella](#), M. Schulz, I. Takimoto Schmiegelow and J. Harms, *The geometry of lunar gravitational wave detection*, arXiv preprint, 3 June 2026, [10.48550/arXiv.2606.04918](https://arxiv.org/abs/2606.04918)

TALKS AND CONFERENCES

- “Quasinormal Ringing of Kerr Black Holes from an Equatorial Plunge”, Invited Speaker at Johns Hopkins University, Baltimore (USA), *October 23rd 2025*
- “Quasinormal Ringing of Kerr Black Holes from an Equatorial Plunge”, XVIII Black Holes Workshop, Lisbon (Portugal), *December 19th 2025*
- “Quasinormal Ringing of Kerr Black Holes from an Equatorial Plunge”, XV ET Symposium, Aachen (Germany), *May 27th 2026*
- “Ringdown modelling of asymmetric mass mergers”, Invited Speaker at University of Nottingham, Nottingham (UK), *September 30th 2026*

VISITING PERIODS

- *01/09/2025 - 28/02/2026*, Johns Hopkins University, Baltimore (USA) as part of PNRR scholarship
- *21/09/2026 - 02/10/2026*, University of Nottingham, Nottingham (UK)